which, by virtue of the relation $A+B+C=\pi$, reduces to

$$\frac{R^2}{2}\{\sin 2A+\sin 2B-\sin 2(A+B)\}.$$

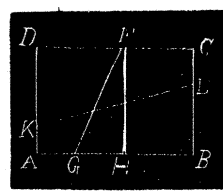
$$\therefore\ \text{Average area}=\frac{R^2}{2}\,\frac{\int_0^{\pi}\int_0^{\pi-A}\{\sin 2A+\sin 2B-\sin 2(A+B)\}dAdB}{\int_0^{\pi}\int_0^{\pi-A}dAdB}=\frac{3R^2}{2\pi}.$$

31. Proposed by B. F. FINKEL, A. M., Professor of Mathematics and Physics in Drury College, Springfield, Missouri.

Find the average length of a line drawn across the opposite sides of a rectangle, length l and breadth b.

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Applied Science, Texarkana College, Texarkana, Arkansas-Texas; and the PROPOSER.

Let $ABCD$ be the rectangle, FG the random line. Let $AB=l$, $BC=b$, $AH=x$, $AG=y$.



Then $FG=\{b^2+(x-y)^2\}^{\frac{1}{2}}$.

The limits of x are 0 and l; of y, 0 and x.

Hence the required average area is

$$\Delta=\frac{\int_0^l\int_0^x\{b^2+(x-y)^2\}^{\frac{1}{2}}dxdy}{\int_0^l\int_0^x dxdy}$$

$$=\frac{2}{l^2}\int_0^l\int_0^x\{b^2+(x-y)^2\}^{\frac{1}{2}}dxdy$$

$$=\frac{1}{l^2}\int_0^l\{x(b^2+x^2)^{\frac{1}{2}}+b^2\log[x+(b^2+x^2)^{\frac{1}{2}}]-b^2\log b\}dx$$

$$=\frac{1}{3l^2}(l^2+b^2)^{\frac{3}{2}}+\frac{b^2}{l}\log\{l+(l^2+b^2)^{\frac{1}{2}}\}-\frac{b^2}{l}\log b-\frac{1}{l^2}(l^2+b^2)^{\frac{1}{2}}-\frac{b^3}{3l^2}+\frac{b}{l^2}.$$

For the line KL, we get, by writing l for b and b for l,

$$\Delta_1=\frac{1}{3b^2}(l^2+b^2)^{\frac{3}{2}}+\frac{l^2}{b}\log\{b+(l^2+b^2)^{\frac{1}{2}}\}-\frac{l^2}{b}\log l-\frac{1}{b^2}(l^2+b^2)^{\frac{1}{2}}-\frac{l^3}{3b^2}+\frac{l}{b^2}.$$

Cor. I. If $l=b$, $\Delta=\frac{1}{3}(2l\sqrt{2})+l\log(1+\sqrt{2})-\frac{1}{l}\sqrt{2}-\frac{1}{3}l+\frac{1}{l}$.

Cor. II. If $l=b=1$, $\Delta=\frac{1}{3}(2-\sqrt{2})+\log(1+\sqrt{2})$, which is the same result as given in *Williamson's Integral Calculus*, page 409.

Also solved by *F. P. MATZ*.